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Computational Study of the Use of Set Trimming for the Globally Optimal Design of Gasketed-Plate Heat Exchangers

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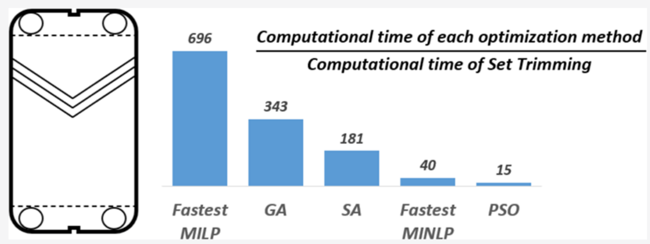

ABSTRACT: In this article, we present for the first time, a globally optimal design procedure of gasketed-plate heat exchangers using a new proposed technique: Set Trimming. Set Trimming is a recently developed optimization technique based on the sequential application of inequality constraints to gradually reduce the search space. This method eliminates several drawbacks that affect other optimization techniques: (i) it guarantees global optimality; (ii) it does not depend on good initial estimates; (iii) it guarantees convergence; and (iv) it does not require any tuning of algorithm parameters. The formulation of the optimization problem corresponds to the minimization of the heat exchanger area or the total annualized cost subjected to pressure drop bounds, flow velocity bounds, and required area constraint. Another contribution of this work is a new analysis of the Set Trimming method by finding the fastest algorithmic alternative through methodical application of each constraint. To validate the conjecture that our Set Trimming method is computationally faster for the design of plate heat exchangers, we compare its performance with results obtained using mixed-integer nonlinear programming (MINLP), mixed-integer linear programming (MILP), particle swarm optimization (PSO), genetic algorithms (GA), and simulated annealing (SA), which showed increased performance by orders of magnitude. These results suggest that Set Trimming can be a useful resource for the solution of design problems when the degrees of freedom are represented by integer variables.

1. INTRODUCTION

Traditionally, gasketed-plate heat exchangers have been employed in industrial sectors such as, dairy products, beverages, etc.[1] The ease of cleaning is a fundamental aspect that justifies the importance of gasketed-plate heat exchangers for the food industry.[2] However, gasketed-plate heat exchangers are sometimes a better option than shell-and-tube exchangers in conventional process plants too.[3] They present several advantages, such as low cost,[1] higher film coefficients,[1−3] less propensity to fouling,[1] low heat losses to the surroundings (no insulation required),[3] and a compact size.[2] Their main limitations are associated with operational restrictions related to the gaskets and more elevated pressure drops, e.g., plate heat exchangers are only usually employed in temperatures below 160−250 °C and pressures up to 25−30 bar, and are not suitable for gas-to-gas applications or highly viscous fluids.[3]

The optimization of the design of gasketed-plate heat exchangers was addressed in the literature by several authors, where the main approach is based on the minimization of the heat transfer surface subjected to pressure drop bounds.[3−10] Another option of the single objective function encompasses capital and operating costs.[4,11−13] The optimization of the design of plate heat exchangers was also addressed in the literature using multiobjective formulations, where different objective functions were investigated simultaneously, e.g., the total cost and effectiveness,[14] heat transfer coefficient and pressure gradient,[15] heat load and number of thermal plates,[16] and overall heat transfer coefficient and pressure drop.[17,18]

Different optimization techniques were explored for the design of gasketed-plate heat exchangers: stochastic methods,[13−17] heuristic and enumeration techniques,[5−7,9] and mathematical programming.[4,10]

Stochastic optimization involves the utilization of randomized algorithms, usually mimicking a natural phenomenon. The main stochastic optimization technique tested for the design optimization of plate heat exchangers was genetic algorithms, usually addressing the multiobjective optimization problem.[13−17]

In turn, mathematical programming uses algorithms that seek the identification of the optimum based on rigorous optimality conditions. Despite the extensive use of mathematical programming for chemical process equipment design, few attempts exist for the design of gasketed-plate heat exchangers: Wang and Súnden,[4] who employed nonlinear programming, and Martins et al.,[10] who employed mixed-integer linear

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programming (MILP), based on a rigorous transformation of the original problem.

Heuristic and enumeration techniques involve exploring the search space through the generation of a sequence of solution alternatives until an optimal solution is reached. For example, Zhu and Zhang[5] solved the design problem using a simple enumeration technique, based on the testing of heat exchangers of crescent size. Screening procedures employing the constraints to reduce the number of candidates were explored by Gut and Pinto[6] and Mota et al.[9] Another alternative of this kind of algorithms is based on a graphical delimitation of the search space that allows the designer to identify the best solution.[7]

The current article explores the utilization of a recently developed technique called Set Trimming for the design of plate heat exchangers. This technique is an extension and formalization of the screening step in the optimization procedure developed by Gut and Pinto,[6] and it was presented in the form of a complete and autonomous optimization technique for equipment design only by Costa and Bagajewicz.[19] Aside from guaranteeing global optimality regardless of the nonlinearity of the model, including the objective function, it is claimed to have excellent computational performance. Indeed, a previous application of Set Trimming for the design of shell-and-tube heat exchangers showed promising results, reducing the time by orders of magnitude compared to alternative solution techniques.[20] This work supplies further evidence of the effectiveness of this approach. Besides, we reiterate that Set Trimming guarantees global optimality, is robust (does not need initial data or help), and is not affected by higher degrees of nonlinearity. Previous attempts to solve the design problem presented limitations in relation to one or more of these issues. Stochastic optimization techniques can avoid local optima, but it is not possible to guarantee that the solution obtained is the global optimum; additionally, the need of careful tuning of the control parameters can affect its robustness. Mathematical programming techniques applied to nonlinear formulations of the design problem using local solvers may not converge or may converge to a poor local optima; on the other hand, the utilization of rigorous linear reformulations (as introduced by Gonçalves et al.[21,22] and outlined by Costa and Bagajewicz[19]) always attains the global optimum but may be slow. Enumeration techniques that are also able to attain the global optimum, but did not explore the full search space,[6,9] are based on exhaustive enumeration of certain design variables, which can be slow.[5,7]

In this article, we illustrate the application of Set Trimming to this problem and compare its performance with mathematical programming (mixed-integer nonlinear programming, MINLP and mixed-integer linear programming, MILP) and stochastic methods (particle swarm optimization, PSO; genetic algorithms, GA; and simulated annealing, SA).

Our article is organized as follows. Section 2 presents the design problem of gasketed-plate heat exchangers; Section 3 presents the nonlinear equations of the thermofluid-dynamic heat exchanger model; Section 4 explores the application of the Set Trimming method for the design problem; Section 5 presents the numerical results, where the Set Trimming performance is compared with mathematical programing and stochastic optimization techniques; and Section 6 presents the conclusions.

2. OPTIMIZATION OF GASKETED-PLATE HEAT EXCHANGERS

The design optimization problem addresses gasketed-plate heat exchangers with Chevron-type plates for the heating/cooling of streams without a phase change. The physical properties of the streams are assumed constant, evaluated at the average of the stream temperatures. The impact of end plates in the temperature profile is assumed negligible. The thermofluid-dynamic model employed to describe the behavior of the heat exchangers is based on Kakaç and Liu,[3] according to the plate dimensions illustrated in Figure 1. It is important

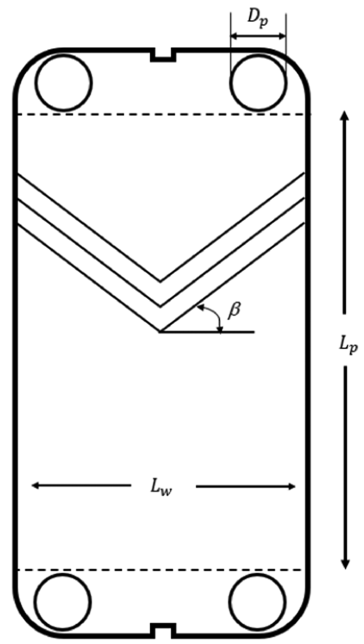

Figure 1. Main dimensions of a Chevron plate.

to observe that the proposed optimization approach is flexible and could be adapted to any other model with the same features, e.g., the same approach can be employed for the design of heat exchangers with streams with a phase change at constant temperature assuming average values of heat transfer coefficients.

The design optimization problem seeks to identify a plate heat exchanger with a minimum area, subjected to flow velocity bounds, pressure drop bounds, and required area constraint.

The design variables that represent each solution candidate are: the total number of plates (N_t), plate size (defined by the plate length, L_p, plate width, L_w, and port diameter, D_p), Chevron angle (β), and number of passes of each stream (N_{ph} and N_{pc}, respectively). Due to the commercial availability and/or physical nature of the design variables, their values must be selected among a set of discrete options. The plate thickness ($\hat{t}$), the surface enlargement factor ($\hat{\varnothing}$), and the mean channel spacing ($\hat{b}$) are fixed parameters associated with the plate type and are not included in the optimization.

The optimization problem corresponds to the minimization of the total area of the plates

$$\min A_{tot} = N_t\, \hat{\varnothing}\, L_w\, L_p \quad (1)$$

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The performance constraints encompass the bounds on flow velocities, available pressure drops, and the minimum excess area required to fulfill the thermal task, as follows

$$\hat{v}_{\min} \leq v \leq \hat{v}_{\max} \quad (2)$$

$$\Delta P_t \leq \Delta \hat{P}_{\text{disp}} \quad (3)$$

$$A \geq \left(1 + \frac{\widehat{A_{\text{exc}}}}{100}\right) A_{\text{req}} \quad (4)$$

where v is the flow velocity in the channels between the plates, $\hat{v}_{\min}$ and $\hat{v}_{\max}$ are the upper and lower flow velocity limits, respectively, ΔP_t is the total pressure drop, $\Delta \hat{P}_{\text{disp}}$ is the available pressure drop, A is the heat transfer area, A_{req} is the required heat transfer area, and $\hat{A}_{\text{exc}}$ is the excess area (expressing a design margin). The heat transfer area does not include the end plates

$$A = (N_t - 2)\,\hat{\varpi}\, L_w\, L_p \quad (5)$$

The constraints represented in eqs 2 and 3 must be applied to both streams.

Alternatively, the heat exchanger design optimization can also be formulated as the minimization of the total annualized cost, including capital and operating costs. In this case, the constraint related to the pressure drop bounds in eq 3 is eliminated and the objective function displayed in eq 1 is substituted by

$$\min \text{TAC} = \hat{r} C_{\text{cap}} + C_{\text{op,h}} + C_{\text{op,c}} \quad (6)$$

where TAC is the total annualized cost, C_{cap} is the capital cost, $C_{\text{op,h}}$ and $C_{\text{op,c}}$ are the operating costs on a yearly basis for the hot and cold streams, and $\hat{r}$ is the annualizing factor. The expression of the annualizing factor is

$$\hat{r} = \frac{\hat{i}(1 + \hat{i})^{\hat{n}}}{(1 + \hat{i})^{\hat{n}} - 1} \quad (7)$$

where $\hat{i}$ is the interesting rate and $\hat{n}$ is the project horizon in years.

The capital cost can be evaluated by

$$C_{\text{cap}} = \hat{a} A_{\text{tot}}^{\hat{b}} \quad (8)$$

where $\hat{a}$ and $\hat{b}$ are model parameters of the cost correlation. The expression for the evaluation of the energy consumption for each stream is given by

$$C_{\text{op}} = \widehat{N_{\text{op}}} \frac{\widehat{p_c}}{10^3} \left(\frac{\Delta P_t \hat{m}}{\hat{\eta} \hat{\rho}}\right) \quad (9)$$

where $\widehat{p_c}$ is the energy price, $\hat{m}$ is the mass flow rate (hot or cold), $\hat{\eta}$ is the pump efficiency, $\hat{\rho}$ is the fluid density (hot or cold), and $\widehat{N_{\text{op}}}$ is the number of operating hours per year.

3. THERMOFLUID-DYNAMIC MODEL

We now present the gasketed-plate heat exchanger model, encompassing thermal and fluid-dynamic equations. The model parameters, which are fixed prior to the optimization (fluid properties, flow rates, temperatures, etc.), are represented with the symbol ∧ on top.

3.1. Thermal Equations. The convective heat transfer coefficient is evaluated by the following correlation

$$Nu = C Re^n \widehat{Pr}^{1/3} \quad (10)$$

where Nu, Re, and $\widehat{Pr}$ are the Nusselt, Reynolds, and Prandtl numbers, respectively, and C and n are model parameters that depend on the Reynolds number and Chevron angle, as shown in Table 1.

Table 1. Model Parameters for the Convective Heat Transfer Coefficient

Chevron angle, β (deg)	Re	C	n
≤30	≤10	0.718	0.349
	>10	0.348	0.663
45	<10	0.718	0.349
	10−100	0.400	0.598
	>100	0.300	0.663
50	<20	0.630	0.333
	20−300	0.291	0.591
	>300	0.130	0.732
60	<20	0.562	0.326
	20−400	0.306	0.529
	>400	0.108	0.703
≥65	<20	0.562	0.326
	20−500	0.331	0.503
	>500	0.087	0.718

The expressions of the dimensionless numbers are

$$Nu = \frac{h \widehat{D_{\text{hyd}}}}{\hat{k}} \quad (11)$$

$$Re = \frac{G_c \widehat{D_{\text{hyd}}}}{\hat{\mu}} \quad (12)$$

$$\widehat{Pr} = \frac{\widehat{C_p} \hat{\mu}}{\hat{k}} \quad (13)$$

where h is the convective heat transfer coefficient, $\widehat{D_{\text{hyd}}}$ is the hydraulic diameter of the flow channel between plates, G_c is the stream mass flux through the channels between plates, $\widehat{C_p}$, $\hat{\mu}$, and $\hat{k}$ are the heat capacity, viscosity, and thermal conductivity of the stream.

The hydraulic diameter can be expressed by

$$\widehat{D_{\text{hyd}}} = \frac{2\hat{b}}{\hat{\phi}} \quad (14)$$

The mass flux through the channels depends on the stream flow rate, the number of channels, and the number of passes of the corresponding stream

$$G_c = \frac{\hat{m}/N_{\text{cp}}}{\hat{b} L_w} \quad (15)$$

where $\hat{m}$ is the stream mass flow rate and N_{cp} is the number of channels between plates per pass. The number of channels per pass is related to the total number of plates and the number of passes

$$N_{\text{cp}} = \frac{(N_t - 1)}{2 N_p} \quad (16)$$

The heat transfer rate equation can be represented by

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Ind. Eng. Chem. Res. 2021, 60, 1746−1755

$$\hat{Q} = UA_{req}\widehat{\Delta T_{lm}}F \tag{17}$$

where $\hat{Q}$ is the heat exchanger heat load, U is the overall heat transfer coefficient, $\widehat{\Delta T_{lm}}$ is the logarithmic mean temperature difference, and F is the correction factor.

The expression of the overall heat transfer coefficient is

$$U = \frac{1}{\frac{1}{h_h} + \widehat{R_{fh}} + \frac{\hat{t}}{\widehat{k_w}} + \widehat{R_{fc}} + \frac{1}{h_c}} \tag{18}$$

where h_h and h_c correspond to the convective heat transfer coefficients calculated through the correlation in eq 6 for the hot and cold streams, respectively, $\widehat{R_{fh}}$ and $\widehat{R_{fc}}$ are the fouling factors for the hot and cold streams, respectively, and $\widehat{k_w}$ is the thermal conductivity of the plate material.

The logarithmic mean temperature difference is given by

$$\widehat{\Delta T_{lm}} = \frac{(\widehat{T_{hi}} - \widehat{T_{co}}) - (\widehat{T_{ho}} - \widehat{T_{ci}})}{\ln\left(\frac{(\widehat{T_{hi}} - \widehat{T_{co}})}{(\widehat{T_{ho}} - \widehat{T_{ci}})}\right)} \tag{19}$$

where $\widehat{T_{hi}}$ and $\widehat{T_{ho}}$ are the inlet and outlet temperatures of the hot stream and $\widehat{T_{ci}}$ and $\widehat{T_{co}}$ are the inlet and outlet temperatures of the cold stream, respectively.

The correction factor depends on the heat exchanger end temperatures and the corresponding configuration related to the number of passes of each stream. For a given problem, the values of the correction factor for each combination of the number of passes of each stream are calculated prior to the optimization through the expressions for effectiveness reported by Kandlikar and Shah[23] using the approach adopted by Souza et al.[24] for air coolers.

3.2 Fluid-Dynamic Equations. The fluid-dynamic model corresponds to the evaluation of the total pressure drop of the hot and cold streams, encompassing the pressure drop through the channels between plates (ΔP_c) and the pressure drop along the port (ΔP_p)

$$\Delta P_t = \Delta P_c + \Delta P_p \tag{20}$$

The pressure drop in the channels is given by

$$\Delta P_c = f\frac{L_p + D_p}{\widehat{D_{hyd}}}\frac{{G_c}^2}{2\hat{\rho}} \tag{21}$$

where f is the friction factor and $\hat{\rho}$ is the density of the fluid. The friction factor is given by

$$f = KRe^m \tag{22}$$

where K and m depend on the Chevron angle and the Reynolds number (Table 2).

The pressure drop in the plate port is given by

$$\Delta P_p = 1.4N_p\frac{{G_p}^2}{2\hat{\rho}} \tag{23}$$

where G_p is the mass flux through the plate port

$$G_p = \frac{\hat{m}}{\left(\frac{\pi {D_p}^2}{4}\right)} \tag{24}$$

Finally, the flow velocity in eq 2 is given by

$$v = \frac{G_c}{\hat{\rho}} \tag{25}$$

Table 2. Model Parameters for the Friction Factor

Chevron angle, β (deg)	Re	K	m
≤30	<10	50.000	1.000
	10−100	19.400	0.589
	>100	2.990	0.183
45	<15	47.000	1.000
	15−300	18.290	0.652
	>300	1.441	0.206
50	<20	34.000	1.000
	20−300	11.250	0.631
	>300	0.772	0.161
60	<40	24.000	1.000
	40−400	3.240	0.457
	>400	0.760	0.215
≥65	<50	24.000	1.000
	50−500	2.800	0.451
	>500	0.639	0.213

4. SET TRIMMING

The independent variables of the problem are purely geometric. In other words, once one set of geometric variables (number of plates, length, width, etc.) is chosen, the problem reduces to a performance evaluation (velocities below bounds, thermal task accomplished, and pressure drop below limits) and an evaluation of the objective function. We start by clarifying that there are two representations of the search space. One is the traditional, which is characterized by each discrete geometric variable being represented by the set of options. The other is called a combinatorial representation, where each element of the space is one combination of all of the geometric variables. Each element of this representation is therefore a candidate solution.

The Set Trimming procedure relies upon a systematic application of the problem inequality constraints to eliminate solution candidates from the combinatorial space.[19] This reduces the cardinality of the set of candidates applying the constraints sequentially one after the other. If all of the constraints are used, then the resultant list of candidates contains all feasible solutions and the global optimal solution can be identified through a simple sorting procedure. This is our case. In other cases, not all of the constraints can be efficiently used and at the end the set left, even though much smaller in size, contains feasible as well as infeasible candidates. Such cases require an extra step; we do not use it here because at the end of the Set Trimming, all our candidates are feasible.

We emphasize that Set Trimming is not equivalent to enumeration, nor any enumeration is involved. Enumeration requires the evaluation of the objective function and constraints of candidate solutions one by one. As we describe in this article, Set Trimming does not operate analyzing individual candidates, but it eliminates sets of candidates using the problem constraints. Computationally, Set Trimming involves set manipulations (dynamic sets in GAMS and Numpy arrays in Python), instead of enumeration loops associated with the analysis of individual candidates. We also add that Set Trimming exhibits simplicity and requires no mathematical programming-based solvers.

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Ind. Eng. Chem. Res. 2021, 60, 1746−1755

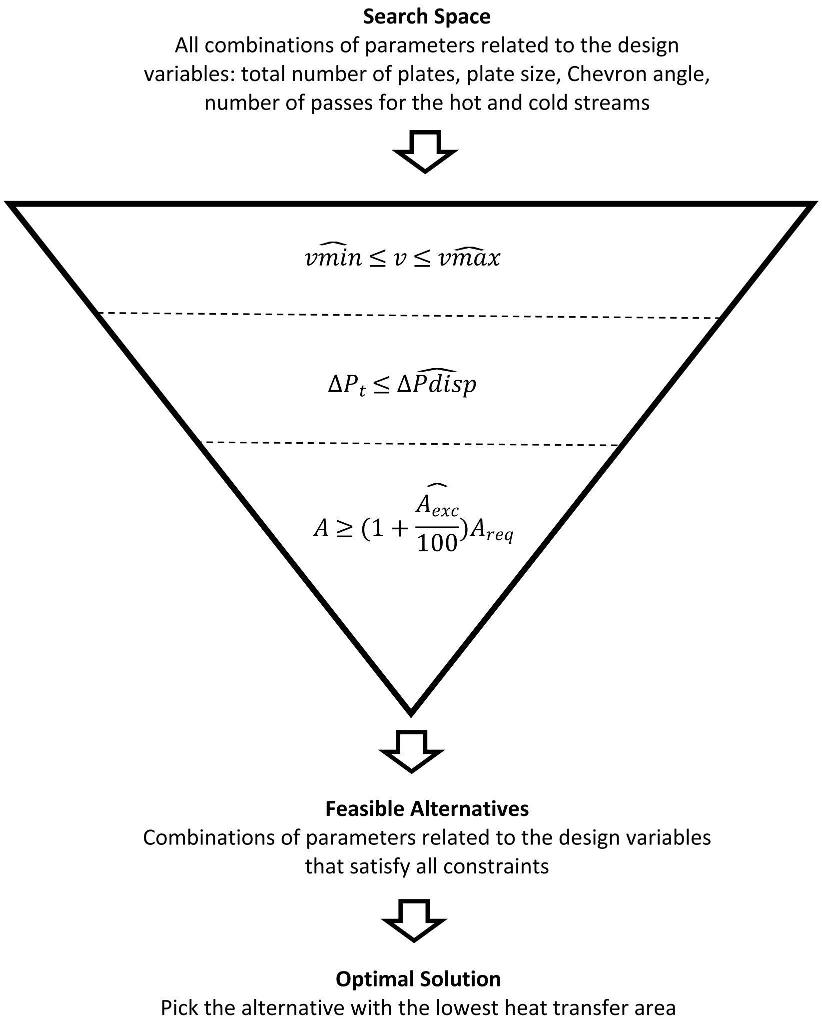

Figure 2. Set trimming procedure.

An additional issue in the application of Set Trimming is the order in which the constraints are applied. Clearly, the performance of the use of one constraint to trim is associated with its ability to cut effectively large quantities of infeasible candidates. The time consumed is another indication of its efficiency. Thus, the overall computational effort can be reduced through the adequate ordering of the constraints. Numerical tests indicated that it is faster to initially test the simpler constraints and test more complex constraints later. In doing so, the more complex constraints, associated with larger computing times, are tested to a smaller number of candidates.

Particularly, in our case, the sequence of constraints is: (1) flow velocity bounds (eq 2), (2) maximum pressure drop (eq 3), and (3) constraint associated with the required area (eq 4). The search procedure is illustrated in Figure 2.

The results shown below contain a novel quantitative analysis of the performance of this sequence of trimmings and a comparison with other alternative ordering to show the importance of an adequate selection of the trimming sequence.

5. RESULTS

The performance of the Set Trimming technique for the design of plate heat exchangers was compared with different optimization algorithms belonging to two classes of methods: mathematical programming and stochastic optimization. The examples are based on design problems originally proposed by Gonçalves et al.[21] for area minimization of shell-and-tube heat exchangers. The complete description of all examples used for testing are present in the Supporting Information. The comparison between Set Trimming and mathematical programming approaches was conducted using Examples 1–5, and the comparison involving stochastic methods was performed on Examples 6–10 (the unique difference between the original Examples 9 and 10 is the fluid allocation, tube-side or shell-side, therefore, to differentiate them here for the investigation of the design of plate heat exchangers, the flow rates of Example 10 were multiplied by two). The search space of the examples is shown in Tables 3 and 4. The information about the dimensions of the plates were obtained from an industrial supplier. The material of the plates has a thickness and thermal conductivity equal to 0.8 mm and 16.2 W/(m^2 K),

Table 3. Values of the Discrete Design Variables

variable	examples 1–5	examples 6–10
total number of plates	62–400 (two by two)	10–800
plate size alternatives (Table 4)	2–6	1–10
Chevron angle (deg)	30, 45, 50, 60, 65	30, 45, 50, 60, 65
hot/cold stream number of passes	1 and 2	1 and 2

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Ind. Eng. Chem. Res. 2021, 60, 1746–1755

Table 4. Dimensions of the Plate Size Alternatives

alternative	plate length, L_p (m)	plate width, L_w (m)	port diameter, D_p (m)
1	0.400	0.125	0.030
2	0.743	0.845	0.300
3	0.978	0.812	0.288
4	1.281	1.200	0.400
5	1.500	1.220	0.350
6	1.835	0.945	0.300
7	2.092	1.200	0.400
8	1.551	0.909	0.285
9	1.845	0.45	0.155
10	1.543	0.812	0.283

respectively. The enlargement factor associated with the corrugation is 1.15. The mean channel spacing is 3 mm. All test runs were executed using a computer with a processor i7-8565U 1.8GHz with 8 GB RAM memory. Additionally, the same problems were also solved for the minimization of the total annualized cost using Set Trimming.

5.1. Area Minimization: Mathematical Programming and Set Trimming. The mathematical programming methods employed in the tests were: mixed-integer nonlinear programming (MINLP) and mixed-integer linear programming (MILP) using two different alternatives of mathematical formulations. The details of the problem formulations for the application of each technique are described in the Supporting Information (MINLP, MILP using a discrete representation of the search space, and MILP using a combinatorial representation of the search space). All of these mathematical programming techniques were tested using the software GAMS.[25] The MINLP approach was explored using two local solvers (SBB[26] and DICOPT[27]) and two global solvers (BARON[28] and ANTIGONE[29]). The MILP solutions were obtained using the solver CPLEX.[30] The Set Trimming technique was also applied using GAMS, employing dynamic sets and without the need to run any solver.

Table 5 contains the optimal heat transfer area attained by each method, where it is possible to observe that Set Trimming

Table 5. Examples 1−5: Optimization Results for Area Minimization

	heat transfer area (m^2)				
example	MINLP using SBB	MINLP using DICOPT	MINLP using ANTIGONE	MINLP using BARON	MILP & Set Trimming
1	399.9	449.9	399.9	399.9	399.9
2	200.7	261.6	206.4	200.7	200.7
3	105.4	147.3	113.1	113.1	105.4
4	635.6	635.6	590.4	590.4	590.4
5	114.1	114.1	114.1	114.1	114.1

attained the global optimum in all examples. Similar results were also obtained by the MILP formulations. The other alternatives did not find the global optimum in at least one of the examples, such that, local optima have areas that are up to 39.7% higher than the global optimum. This illustrates the importance of global optimality for design problems. The details of each globally optimal solution are depicted in the Supporting Information.

Table 6 displays the elapsed time associated with each solution obtained using the different approaches investigated. The analysis of these computational times indicates that Set Trimming was the fastest option, taking between 1.1 and 10.0% of the elapsed time of the fastest mathematical programming alternative, MINLP using SBB. An equivalent analysis only considering the time consumed by the solvers of the mathematical programming approaches does not present significant differences (see the Supporting Information). Table 6 also shows the times for exhaustive enumeration, which is surprisingly smaller than the time employed by mathematical programming, yet not faster than Set Trimming. This reinforces the notion that Set Trimming is not equivalent to exhaustive enumeration.

5.2. Area Minimization: Stochastic Methods and Set Trimming. Set Trimming was also compared with three alternatives of stochastic methods: particle swarm optimization (PSO), genetic algorithms (GA), and simulated annealing (SA). All of these methods were run in Python. The PSO method was tested using the routine available in the module PySwarms[31] using a rounding off procedure to handle the discrete variables.[32] The GA runs employed the module genetic algorithm[33] according to Bozorg-Haddad et al.[34] The SA version employed corresponds to the function dual annealing[35] available in the Scipy module[36] associated with a round off procedure. In all methods, the constraints were handled by the stochastic algorithms through the insertion into the objective function of penalty terms

$$f = A_{tot} + \sum_i (\hat{a} + \hat{b}\Delta g_i) \tag{26}$$

where $\hat{a}$ = 2348.06, $\hat{b}$ =4696.12, and Δg_i is the violation of the constraint *i*. Table 7 displays the set of control parameters employed in the runs of each stochastic method. To allow a more direct comparison of the performances, the Set Trimming technique was also implemented in Python, using Numpy routines.

The area of the heat exchangers obtained using Set Trimming and the stochastic methods are displayed in Table 8. Due to the randomized nature of the stochastic methods, they were tested through 10 independent runs, therefore, the results displayed in Table 8 contain the best solution and the percentage of the runs where this solution was found. The Supporting Information contains the complete description of the global optimum solutions.

The comparison of the results in Table 8 indicates that the Set Trimming always identified the least cost alternative corresponding to the global optimum. The global optimum of each problem was also attained by the stochastic methods (with one exception), but not in all runs. For example, the percentage of success to reach the global optimum of the stochastic alternative with best performance, PSO, varied between 60 and 90%.

Table 9 presents the elapsed time associated with each method. The values reported for the stochastic methods correspond to the average of the 10 runs. Table 9 also shows the times for exhaustive enumeration, which is very slow. Again, this reinforces the notion that Set Trimming is not equivalent to exhaustive enumeration.

Despite eventual differences related to how the different methods were computationally implemented in Python, the results displayed in Table 9 indicate clearly that Set Trimming is faster than all stochastic alternatives. The elapsed times of the Set Trimming runs are only 4.7−8.6% of the corresponding elapsed times of PSO.

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Ind. Eng. Chem. Res. 2021, 60, 1746−1755

Table 6. Examples 1−5: Elapsed Time in the Area Minimization Runs

	elapsed time (s)							
example	MINLP (SBB)	MINLP (DICOPT)	MINLP (BARON)	MINLP (ANTIGONE)	MILP DR[a]	MILP CR[b]	exhaustive enumeration	Set Trimming
1	12.8	950.0	24.5	304.5	631.7	64.8	0.41	0.24
2	8.4	2652.1	48.0	314.3	3106.8	180.4	0.40	0.23
3	5.0	22.2	29.5	105.4	420.3	264.2	0.42	0.20
4	10.4	413.1	20.5	36.5	1127.2	9.9	0.50	0.11
5	1.7	0.95	9.9	1.4	695.1	141.9	0.45	0.17
average	7.66	807.7	26.5	152.4	1196.2	132.2	0.44	0.19

[a]DR, discrete representation of the search space. [b]CR, combinatorial representation of the search space.

Table 7. Examples 6−10: Control Parameters of the Stochastic Methods

method	parameter
PSO	number of particles = 100
	inertia weight = 0.5
	self-confidence factor = 1.5
	swarm confidence factor = 1.75
	maximum number of iterations = 300
GA	population size = 50
	crossover probability = 0.8
	mutation probability = 0.1
	elitism ratio = 0.02
	fraction of the population filled by members of the previous generation = 0.02
	crossover type = uniform
	maximum number of iterations = 300
SA	initial temperature = 5230
	parameter for visiting distribution = 2.62
	parameter for acceptance distribution = −5.0
	reannealing control parameter = 2×10^{-5}
	number of global search iterations = 1000

Table 8. Examples 6−10: Optimization Results for Area Minimization

	optimal heat transfer area, m^2 (fraction of runs where the global optimum was found)			
example	PSO	GA	SA	Set Trimming
6	238.3 (70%)	238.3 (20%)	238.3 (90%)	238.3
7	137.9 (90%)	137.9 (40%)	137.9 (90%)	137.9
8	713.9 (90%)	713.9 (20%)	713.9 (30%)	713.9
9	242.0 (60%)	252.8 (0%)	242.0 (70%)	242.0
10	496.7 (80%)	496.7 (10%)	496.7 (70%)	496.7

Table 9. Examples 6−10: Elapsed Time in the Area Minimization Runs

	elapsed time (s)				
example	PSO	GA	SA	exhaustive enumeration	Set Trimming
6	0.79	15.7	8.42	94.8	0.037
7	0.84	16.7	8.11	88.2	0.060
8	0.67	16.0	9.00	94.1	0.034
9	0.70	16.6	8.60	98.3	0.057
10	0.57	16.3	8.65	97.1	0.049
average	0.714	16.26	8.56	94.5	0.0474

5.3. Total Annualized Cost Minimization: Set Trimming.

The solution of the 10 optimization examples were also conducted using Set Trimming for minimization of the total annualized cost. The values of the parameters employed for the evaluation of the objective function were reported by Hajabdollahi et al.[13] The parameters $\hat{a}$ and $\hat{b}$ for the evaluation of the capital cost are equal to 635.14 and 0.778, respectively. The energy price is 0.15 \$/kWh, the pump efficiency is 0.6, the interest rate is 0.1 for a project horizon of 10 years, and the number of operating hours per year is 7500 h/y.

The optimization results are displayed in Table 10, which contains the area, the total annualized cost, and the corresponding elapsed time attained through the Set Trimming method. The elapsed times of the runs of Examples 1−5 were obtained using GAMS and the corresponding times of Examples 6−10 were obtained using Python. The details of each optimal solution are depicted in the Supporting Information.

Table 10. Optimization Results for the Total Annualized Cost Minimization (10 Examples)

example	area	TAC (\$/y)	elapsed time (s)
1	707.1	29 667	0.23
2	241.1	13 393	0.13
3	142.5	8367	0.20
4	797.7	56 529	0.18
5	170.4	10 742	0.098
6	461.4	31 174.5	0.027
7	225.6	10 950.3	0.030
8	825.7	60 492.6	0.033
9	540.9	23 681.4	0.074
10	1079.6	52 532.3	0.051

The heat exchanger areas of the solutions displayed in Table 10 are higher than the results depicted in Tables 5 and 8. This pattern is expected because the minimization of the total annualized cost seeks a trade-off between capital and operating costs, i.e., the increase of the heat transfer area in the solutions presented in Table 10 allowed a decrease of the flow velocities that brought a reduction of the pressure drop to minimize the new objective function.

5.4. Analysis of the Set Trimming Ordering.

The sequence of the trimmings associated with the different constraints is an important aspect to provide the best performance for the optimization method. Aiming at supporting the trimming order employed in the current article to solve the plate heat exchanger design problem, a quantitative analysis is discussed below.

To find out the power of each trimming, the trimming procedure is applied for each constraint separately to the initial set of candidates. Based on the number of candidates trimmed by each constraint and the time taken, the number of

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Ind. Eng. Chem. Res. 2021, 60, 1746−1755

candidates eliminated per second for each trimming is computed. This result corresponds to the power of that particular trimming. It is reasonable to suppose that the fastest version of the trimming sequence is associated with a decreasing order of the trimming power.

This test was carried out using the Set Trimming implementation in GAMS applied to the area minimization of Example 1, but associated with the search space employed for Examples 6−10, described in Tables 3 and 4 (it is important to notice that the search space employed in this test is higher than the one employed in the performance comparisons in Table 6, 158 200 × 17 000). The corresponding results are displayed in Table 10.

The results in Table 11 show that the trimming sequence employed in this article is coherent with the relative values of the trimming power, i.e., flow velocity > maximum pressure drop constraint > required area constraint. This order of the trimming can solve the problem in 0.69 s. Aiming at illustrating the potential performance loss due to inadequate ordering of the trimmings, the optimization using the reverse order of the trimmings corresponds to a computational time of 1.89 s, i.e., an increase of 173%. It is interesting to observe the larger computational solving time efficiency of the Set Trimming procedure as compared to other solution alternatives: even solving the problem with a number of alternatives almost 10 times higher and an inadequate selection of the trimming sequence, the resultant elapsed time is only 15% of the fastest mathematical programming option analyzed in Table 6.

Table 11. Quantitative Analysis of the Trimming Associated with Each Constraint

alternative	bounds on flow velocity	maximum pressure drop constraint	required area constraint
initial candidates	158 200	158 200	158 200
remaining candidates	33 530	61 078	73 193
candidates eliminated	124 670	97 122	85 007
elapsed time (s)	0.339	1.309	1.367
candidates eliminated/s	367 758	74 195	62 185

6. CONCLUSIONS

This article presented the application of Set Trimming for the design of gasketed-plate heat exchangers. Set trimming is a recently developed optimization technique based on a gradual reduction of the search space through the successive application of the set of inequality constraints.

Based on a sample of different design examples, a comparison of the performance of Set Trimming with several mathematical programming and stochastic methods indicated that Set Trimming always attained the global optimum and it was the fastest option, solving the optimization problems in a fraction of the time consumed by all of the other alternatives tested.

Optimal solutions were provided considering the minimization of the heat transfer area and the minimization of the total annualized cost. The comparison of the results showed that the solutions associated with the total annualized cost optimization usually presented lower values of pressure drops, according to the expected trade-off between the capital and operating costs. The higher values of pressure drops associated with the area minimization solutions are linked to higher flow velocities, which aimed at the increase of the heat transfer coefficients.

The good performance of Set Trimming applied to the design of plate heat exchangers indicates that this optimization approach can be a useful resource for the design of the chemical process equipment, particularly for problems where the degrees of freedom are represented by discrete variables. Although we have several examples where Set trimming is faster than other approaches, aside from the evidence presented in this article,[20] the conjecture that it is always faster for a large class or type of problems cannot be confirmed. It will take several more cases to establish a pattern, and it is likely that no number of cases will be enough to generalize the conjecture.

Future work associated with the design optimization of gasketed-plate heat exchangers will involve a more rigorous mathematical modeling based on the numerical discretization of the differential equations of the energy balances. This will overcome the accuracy limitations of the LMTD method (e.g., large variations of the physical properties with temperature). In these cases, Set Trimming can still be used with some conceptual additional developments explored in a follow-up paper.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.iecr.0c04751.

Design examples; optimal results; MINLP formulation; MILP formulation—discrete representation of the search space; and MILP formulation—combinatorial representation of the search space (PDF)

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Notes

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NOMENCLATURE

Parameters

$\widehat{A_{exc}}$ excess area (%)
$\hat{b}$ mean channel spacing (m)
$\widehat{C_p}$ heat capacity (J/(kg·K))
$\widehat{D_{hyd}}$ hydraulic diameter of the channel plate (m)
$\hat{k}$ thermal conductivity of the stream (W/(m·K))
$\widehat{k_w}$ thermal conductivity of the plate material (W/(m·K))
$\hat{m}$ stream mass flow rate (kg/s)
$\widehat{N_{op}}$ number of operating hours per year (h/y)
$\widehat{P_c}$ energy price ($/kWh)
$\widehat{Pr}$ Prandtl number
$\hat{Q}$ heat load (W)
$\hat{r}$ annualizing factor (y^{-1})
$\widehat{R_f}$ fouling factor (m^2·K/W)
$\hat{t}$ plate thickness (m)
$\widehat{T_{hi}}$ inlet temperature of the hot stream (°C)
$\widehat{T_{ho}}$ outlet temperature of the hot stream (°C)
$\widehat{T_{ci}}$ inlet temperature of the cold stream (°C)
$\widehat{T_{co}}$ outlet temperature of the cold stream (°C)
$\widehat{v_{max}}$ flow velocity upper bound (m/s)
$\widehat{v_{min}}$ flow velocity lower bound (m/s)
$\Delta\hat{P}_{disp}$ available pressure drop (Pa)
$\widehat{\Delta T_{lm}}$ logarithmic mean temperature difference (°C)
$\hat{\eta}$ pump efficiency
$\hat{\varpi}$ enlargement factor
$\hat{\mu}$ viscosity of the stream (Pa·s)
$\hat{\rho}$ stream density (kg/m^3)

VARIABLES

A heat transfer area (m^2)
A_{tot} total area of the plates (m^2)
A_{req} required heat transfer area (m^2)
C model parameter
C_{cap} capital cost ($)
C_{op} operating costs ($/y)
D_p port diameter (m)
F LMTD correction factor
f friction factor (dimensionless)
G_c mass flux through the plate channels (kg/(m^2·s))
G_p mass flux through the plate port (kg/(m^2·s))
h convective heat transfer coefficient (W/(m^2·K))
h_c convective heat transfer coefficient of the cold stream (W/(m^2·K))
h_h convective heat transfer coefficient of the hot stream (W/(m^2·K))
K model parameter
m model parameter
n model parameter
N_{cp} number of channels between the plates per pass
N_p number of passes of the hot/cold streams
N_{pc} number of passes of the cold stream
N_{ph} number of passes of the hot stream
N_t total number of plates
Nu Nusselt number (dimensionless)
L_p plate length (m)
L_w plate width (m)
Re Reynolds number (dimensionless)
TAC total annualized cost ($/y)
U overall heat transfer coefficient (W/(m^2·K))
V flow velocity in the channels between plates (m/s)
β Chevron angle (deg)
ΔP_t total pressure drop (Pa)
ΔP_c pressure drop through the channels between plates (Pa)
ΔP_p pressure drop of the flow through the port (Pa)

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